



IMPROVEMENT OF MEDICATION DELIVERY TRICOPTER STABILITY USING ACKERMAN METHOD

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ABSTRACT

Unmanned aerial vehicles (UAVs) have been a subject of interest across many areas of study. Tricopter is a multirotor with three rotors in Y or T configuration for flight operations; capable of tilting with the aid of a servo motor. The study designed stability controller for a developed medication delivery tricopter to enhanced community-based healthcare system in Nigeria. The study adopted a control law based on Ackerman's method to improve the tricopter's settling time, maximum overshoot and damping factor; through a linearized mathematical model. Desired poles were implemented in MATLAB simulink environment to design a suitable controller for the system in accordance with predefined parameters. The output revealed no overshoots along X and Y axes at settling time of 3s, but overshoot along Z axis by 50% at settling time of 2s. Conclusively, the steady state error was zero and it was observed that the medication delivery tricopter response was made stable with the aid of a controller. The result shows that the tricopter model can be made effectively stable and reliable in operation with the use of a controller.

Keywords: *Tricopter, settling time, overshoot and damping factor*

INTRODUCTION

Multicopters are increasingly being deployed for different operations in various fields of endeavors such as military, security and logistics. The study adopts a control model to improve the stability of a tricopter, hence seeking to enhance its use for medication delivery in Nigeria. The stability and availability requires improvement as investigated; these lies on multi-discipline contributions such as electronics, aeronautics, signal process, control and embedded system. The current trend in Unmanned Aerial Vehicles (UAVs) is the development of small flying machines with forward flight capacity as well as hovering (Salazar-Cruz *et al*, 2007). These small flying machines are referred to as multicopters or multirotors (number of rotors). The tricopter- a class of UAV(no onboard pilot) is a multicopter with three rotors. Tricopter simplifies the complex mechanics of the conventional helicopter while delivering high levels of maneuverability in flight control. The control of the multicopter could be achieved by varying the relative speed of each rotor thereby changing the torque and thrust. For the case of odd number rotors like in a tricopter, the torque and thrust are balanced by including a servo control mechanism to the tail rotor. Consequently, a more complicated controller output design is required for maneuvering command compared with quad-rotor system which triggers motion by adjusting the angular speed of its rotor pairs (Yoon, *et al.*, 2013). The study is based on dynamical analysis and established a mathematical model for the tricopter. The study developed a control model to aid the tricopter's robustness and stability using the Proportional-plus-Integral-plus-Derivative Control (PID) and Linear Quadratic Gaussian or Regulator (LQG / LQR) method. The orientation of tricopter operations is based on the application of Newton's second law in consideration of Euler angles such as analysis of fixed frame can derive the roll(ϕ), pitch(θ), and yaw(ψ) angles of the tricopter upon rotation along the x, y, and z axis respectively (Sergio, *et al.*, 2009). Aside this, the study deployed Ackerman's method to predict developed tricopter's stability for its system availability.

MATERIALS AND METHODS

Mathematical Model of the System

Linearization of the adopted Mathematical Model

The operational mode of a tricopter required a feedback control system to enable coordinial stability; this was achieved with support of a controller. A controller functions by comparing the feedback result with the reference value and tries to compensate for

the error continuously to the minimal. The controller design is based on input and output of the connected plant. This is described by the mathematical model of the plant. Controller design procedures are based on linear mathematical model such as root-locus method, bode plots, diagram method and the pole placement method.

The study modelled stability mathematical indices through dynamics of rotation velocities (Apena, 2017), as given in Equations (1a, 1b and 1c) for a medication delivery tricopter by pole placement method and linear controller.

$$\dot{\omega}_{x_b} = \frac{\bar{T}_1 l_1 - \bar{T}_2 l_2 - (l_{x_b x_b} - l_{y_b y_b}) * \omega_{y_b} * \omega_{z_b}}{l_{x_b x_b}} \dots (1a)$$

$$\dot{\omega}_{y_b} = \frac{(\bar{T}_1 + \bar{T}_2) l_3 - \bar{T}_3 l_4 - (l_{x_b x_b} - l_{y_b y_b}) * \omega_{x_b} * \omega_{z_b}}{l_{y_b y_b}} \dots (1b)$$

$$\dot{\omega}_{z_b} = \frac{\bar{H}_1 l_1 - \bar{H}_2 l_2 - \bar{H}_3 l_3 - (l_{y_b y_b} - l_{x_b x_b}) * \omega_{x_b} * \omega_{y_b}}{l_{z_b z_b}} \dots (1c)$$

where H1, H2 and H3 are the torque forces of the three blades, (\bar{T}_1), (\bar{T}_2) and (\bar{T}_3) are the thrust forces of each blade, l is the distance between each motor and the reference point, ω is the angular velocity of each blade, $\dot{\omega}$ is the angular acceleration in the given axis. The subscript 'b' means that the values are taken with reference to the body of the tricopter (the frame), the number indicates the blade involved and the x, y, z subscript indicates the axis.

Thus, the model was simplified and linearized based on some assumptions (Nelson 1997). The model consists of deviations around steady flight conditions; as a result, multiplication of angular velocity components can be omitted and expressed in Equations (2a, 2b and 2c)

$$s\omega_{x_b}(s) = \frac{\bar{T}_1(s)l_1 - \bar{T}_2(s)l_2}{l_{x_b x_b}} \dots (2a)$$

$$s\omega_{y_b}(s) = \frac{[(\bar{T}_1(s) + \bar{T}_2(s))l_3 - \bar{T}_3(s)l_4]}{l_{y_b y_b}} \dots (2b)$$

$$s\omega_{(z_b)}(s) = \frac{\bar{H}_1(s)l_1 - \bar{H}_2(s)l_2 - \bar{H}_3(s)l_3}{l_{z_b z_b}} \dots (2c)$$

Also, making the orientation angles to be equal to the angular velocity, expressed in equation (3), a substitution can be made into Equation (2), resulting in Equations (4a-4c)

$$s \begin{pmatrix} \phi(s) \\ \theta(s) \\ \psi(s) \end{pmatrix} = \begin{pmatrix} \omega_{x_b}(s) \\ \omega_{y_b}(s) \\ \omega_{z_b}(s) \end{pmatrix} \dots\dots\dots 3$$

$$s^2 \phi(s) = \frac{\vec{T}_1(s)l_1 - \vec{T}_2(s)l_2}{I_{x_b x_b}} \dots\dots\dots (4a)$$

$$s^2 \theta(s) = \frac{(\vec{T}_1(s) + \vec{T}_2(s))l_3 - \vec{T}_3(s)l_4}{I_{y_b y_b}} \dots\dots\dots 4b$$

$$s^2 \psi(s) = \frac{\vec{H}_1(s)l_1 - \vec{H}_2(s)l_2 - \vec{H}_3(s)l_3}{I_{z_b z_b}} \dots\dots\dots 4c$$

Considering orientation angles the subject of formulais adjusted to obtain Equations (5a-5c)

$$\phi(s) = \frac{\vec{T}_1(s)l_1 - \vec{T}_2(s)l_2}{s^2 I_{x_b x_b}} \dots\dots\dots 5a$$

$$\theta(s) = \frac{(\vec{T}_1(s) + \vec{T}_2(s))l_3 - \vec{T}_3(s)l_4}{s^2 I_{y_b y_b}} \dots\dots\dots 5b$$

$$\psi(s) = \frac{\vec{H}_1(s)l_1 - \vec{H}_2(s)l_2 - \vec{H}_3(s)l_3}{s^2 I_{z_b z_b}} \dots\dots\dots 5c$$

Furthermore, the transfer function of the tricopter actuators (motors) as derived by Oguntoyinbo(2009) can be made proportional by applying a proportionality coefficient. Reducing the brushless Direct Current Motor (DCM) equation to a 1st order transfer function, we have

$$u = k_{P\text{PWM}} * u_{\text{max}} \dots\dots\dots 6$$

$$G(s) = \frac{1}{T_m \cdot s + 1} \dots\dots\dots 7$$

where u is input voltage to BLDC, u_{max} is the maximum input voltage of the BLDC applied to the power bridge and k_{PWM} is the percentage Pulse-Width Modulation (PWM). The time delay is neglected and the bridge is assumed a continuous element.

Bresciani (2008) suggested that the inductive term could be neglected due to the following assumption:

- Most of the motors used in robotics show small inductance to construction optimization.
- The pole (response time) of the electrical part is always much faster than the mechanic one, therefore the speed of the overall system will be determined just by the slowest contribution.
- It is much easier to solve a first order differential equation rather than a second order one.

Thus, the rotor thrust force and hub force are simplified as shown in Equation 8(a-b),

$$T(s) = b_T * \frac{1}{T_m \cdot s + 1} * k_{\text{PWM}} * u_{\text{max}} \dots\dots\dots 8a$$

$$\frac{T(s)}{k_{\text{PWM}}} = b_T * \frac{1}{T_m \cdot s + 1} * u_{\text{max}} = \frac{K_T}{T_m \cdot s + 1} \dots\dots\dots (8b)$$

Similarly,

$$H(s) = b_H * \Omega^2(s); \dots\dots\dots 9$$

$$H(s) = b_H * \frac{1}{T_m \cdot s + 1} * k_{\text{PWM}} * u_{\text{max}} \dots\dots\dots (10)$$

$$\frac{H(s)}{k_{\text{PWM}}} = b_H * \frac{1}{T_m \cdot s + 1} * u_{\text{max}} = \frac{K_H}{T_m \cdot s + 1} \dots\dots\dots (11)$$

Assuming, the motor dynamics is small compared to the overall system dynamics, the equation is further simplified as:

$$\frac{T(s)}{k_{\text{PWM}}} = K_T \dots\dots\dots (12)$$

$$\frac{H(s)}{k_{\text{PWM}}} = K_H \dots\dots\dots (13)$$

Substituting Equations 12 and 13 into Equation 8a and adopting equal lengths $l_1=l_2=l_3=l_4$, we have

$$\frac{\phi(s)}{k_{\text{PWM1}}(s) - k_{\text{PWM2}}(s)} = \frac{K_T * l}{s^2 I_{x_b x_b}} = \frac{K_\phi}{s^2}$$

$$\frac{\theta(s)}{k_{\text{PWM1}}(s) + k_{\text{PWM3}}(s) - k_{\text{PWM4}}(s)} = \frac{K_T * l}{s^2 I_{y_b y_b}} = \frac{K_\theta}{s^2} \dots\dots\dots (14)$$

$$\frac{\psi(s)}{k_{\text{PWM1}}(s) - k_{\text{PWM2}}(s) - k_{\text{PWM3}}(s)} = \frac{K_H * l}{s^2 I_{z_b z_b}} = \frac{K_\psi}{s^2}$$

$$\text{where } K_\phi = \frac{K_T * l}{I_{x_b x_b}}, K_\theta = \frac{K_T * l}{I_{y_b y_b}}, K_\psi = \frac{K_H * l}{I_{z_b z_b}}$$

The tricopter translational flight dynamics was examined and simplified by neglecting the drag forces. thus

$$r_x(s) = \frac{1}{s^2} \left(\frac{(\cos\phi(s) * \sin\theta(s) * \cos\psi(s) + \sin\phi(s) * \sin\psi(s))}{m} * T(s) \right)$$

$$r_y(s) = \frac{1}{s^2} \left(\frac{(\cos\phi(s) * \sin\theta(s) * \sin\psi(s) - \sin\phi(s) * \cos\psi(s))}{m} * T(s) \right) \dots\dots\dots (15)$$

$$r_z(s) = \frac{1}{s^2} \left(\frac{(\cos\phi(s) * \cos\theta(s))}{m} * T(s) - g \right)$$

The study adopted Caughey (2011) assumptions to consider the system using dynamic equations;

$$\cos\phi \approx 1, \quad \cos\theta \approx 1, \quad \cos\psi = \cos\psi$$

$$\sin\phi \approx \phi, \quad \sin\theta \approx \theta, \quad \sin\psi = \sin\psi, \quad \dots\dots\dots(16)$$

Applying equation (16) to equation (15), the linearized translation flight dynamics becomes

$$r_x(s) = \frac{1}{s^2} \left(\frac{(\theta(s) * \cos\psi(s) + \phi(s) * \sin\psi(s))}{m} * T(s) \right)$$

$$r_y(s) = \frac{1}{s^2} \left(\frac{(\theta(s) * \sin\psi(s) - \phi(s) * \cos\psi(s))}{m} * T(s) \right) \dots\dots\dots (17)$$

$$r_z(s) = \frac{1}{s^2} \left(\frac{1}{m} * T(s) - g \right)$$

where, $T(s) = K_T * (k_{PWM1}(s) + k_{PWM2}(s) + k_{PWM3}(s))$

Therefore, this translation linearises the flight dynamics to a simplified 2nd order system. The attitude and altitude dynamics are derived as given by equation (14) and (17) respectively. Subsequently, a 2nd order controller is needed to stabilize the tricopter flight conditions.

Pole-Placement method: Ackerman approach

The poles placement method refers to as the control method; thus the poles of the characteristics equation of the mathematical model are inter-changed by a designed controller based. Alexander (2013) stated that has limitation on settling time, maximum overshoot and steady state error. This is due to system's sensitivity to physical parameter such direction of wind and drift; thus a state equation of an n-order is applicable as shown in equation 18. The advantage of pole placement method is that the controller can be expanded to an optimal or adaptive controller. The disparity of pole placement method to Ackerman approach is adopted in this study to achieve optimal simplicity.

The state equations of an n-order system in control canonical form are:

$$\dot{x}(t) = A * x(t) + B * u(t)$$

$$y(t) = C * x(t) + D * u(t) \dots\dots\dots(18)$$

where $A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ $C = [b_0 \ b_1 \ b_2 \ \dots \ b_{n-1}]$, $D = 0$.

The Ackerman method approach determines coefficient K that would replace the existing coefficients to make the roots of the closed-loop system to the desired poles. Consequently, the study considered regulator control, input to the system (zero). Thus, the control input $u(t)$ consists only of feedback signals and can be described as:

$$u(t) = -Kx(t), \quad \dots\dots\dots(19)$$

Substituting the control input equation (19) into equation (18) we have:

$$\dot{x}(t) = (A - B * K)x(t) = A_{cl}x(t) \quad \dots\dots(20)$$

Therefore, the close loop matrix A_{cl} can be written in matrix form as

$$A_{cl} = (A - BK) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} * [K_1 \quad K_2 \quad \dots \quad K_{n-1} \quad K_n] =$$

$$= \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 - K_1 & -a_1 - K_2 & -a_2 - K_3 & \dots & -a_{n-1} - K_n \end{bmatrix} \quad \dots\dots\dots(21)$$

The gain matrix K can be derived as:

$$K = [0 \quad 0 \quad \dots \quad 0 \quad 1] * [B \ A * B \ \dots \ A^{n-2} * B \ A^{n-1} * B]^{-1} * \alpha_c(A) \quad \dots\dots\dots(22)$$

where, $\alpha_c(A)$ is matrix polynomial formed with coefficients of the desired characteristics equation $\alpha_c(s)$. The desired closed-loop characteristic equation can be described as

$$\alpha_c(s) = s^n + \alpha_{n-1} * s^{n-1} + \dots + \alpha_1 * s + \alpha_0 = |sI - (A - BK)| =$$

$$= (s + \mu_1) * (s + \mu_2) \dots (s + \mu_n) = 0, \quad \dots\dots\dots(23)$$

where $-\mu_1 \dots -\mu_n$ are the desired poles.

Matrix polynomial $\alpha_c(A)$ is obtained as

$$\alpha_c(A) = A^n + \alpha_{(n-1)} * A^{(n-1)} + \dots + \alpha_1 * A + \alpha_0 * I, \quad \dots\dots\dots(24)$$

where, I is an identity matrix of dimensions equal to that of A.

Pole placement method entails three basic steps. First step is to obtain a linear continuous time invariant mathematical model of the system. The second step is to determine desired poles based on the given system performance parameters such as the settling time, rise time, overshoot and damping factor. The final step is to predict feedback coefficients that change poles of the mathematical model to the desired poles system as expressed in equation (24).

Desired poles for 2nd order system

The characteristic equation of a system defines its stability and frequency response. The choice of desired poles depends on the required performance parameter of the system. Philips and Harbor (2000), described simplest way of choosing poles for 2nd order system and transfer function should be represented as

$$G(s) = \frac{\omega_n^2}{s^2 + 2 * \xi * \omega_n * s + \omega_n^2} \quad \dots\dots\dots(25)$$

where, ω_n is the natural frequency and ξ is the damping ratio.

The response of a transfer function to a step input labelled with its system parameters is described by Dorf

and Bishop (2007), shown in figure 1. The parameters shown include: rise time, peak time, overshoot, settling time, steady state error. Often, for design purposes a minimum settling time and overshoot is required to enhance system stability and effective functionality.

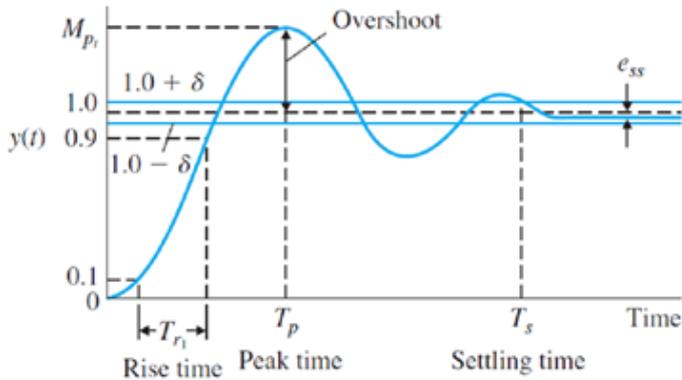


Figure 1: Step response of a second order system (Dorf & Bishop, 2007)

The study considered percentage overshoot %Mp with respect to damping ratio (ξ), while the settling time T_s is dependent on the natural frequency ω_n , expressed in equation (26) and (27).

$$\%M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} * 100 \quad \dots\dots\dots (26)$$

$$T_s = \frac{4}{\xi * \omega_n} \quad \dots\dots\dots (27)$$

Considering, the damping ratio (ξ) and natural frequency ω_n as subjects of the equation,

$$\xi = \frac{\left(\frac{1}{\pi} * \ln \frac{\%M_p}{100}\right)}{\sqrt{1 + \left(\frac{1}{\pi} \ln \frac{\%M_p}{100}\right)^2}} \quad \dots\dots\dots (28)$$

$$\omega_n = \frac{4}{\xi * T_s} \quad \dots\dots\dots (29)$$

Based on ξ and ω_n the desired poles can be determined as

$$s_{1,2} = -\xi * \omega_n \pm j * \omega_n * \sqrt{1 - \xi^2} \quad \dots\dots (30)$$

The control of the medication delivery tricopter requires very fast response with minimal overshooting just like most control systems. The optimal damping ratio ξ for this behavior is 0.707 with a corresponding maximum overshoot of 4.32%. The settling time should be close to zero causing the natural frequency to be at infinity, but this is not practical in real systems due to the nature of topography. Thus, the settling

time $T_s=1.131s$ is chosen. The poles are obtained from equation(30) from the chosen parameters to be:

$$s_{1,2} = -3.5367 \pm j3.5363 \quad \dots\dots\dots (31)$$

The poles were obtained using the MATLAB functions where inputs are settling time and maximum overshoot and the outputs are the desired poles.

RESULTS AND DISCUSSION

Controller Design and Implementation

Attitude Control

The study used MATLAB R2015a SIMULINK to mimic model and designed a controller as shown below with respect to derived models. The mathematical model from equation (14) is a second order equation which has a general form,

$$G(s) = \frac{b_1 * s + b_0}{s^2 + a_1 * s + a_0} \quad \dots\dots\dots 32$$

The designed tricopter revealed a state space representation of canonical form 2nd order system as described by Alexander (2013). This is mathematically represented as

$$\begin{aligned} \dot{x}(t) &= A * x(t) + B * u(t) \\ y(t) &= C * x(t) \end{aligned} \quad \dots\dots\dots (33)$$

where, $A = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [b_0 \ b_1]$.

The system can be represented as:

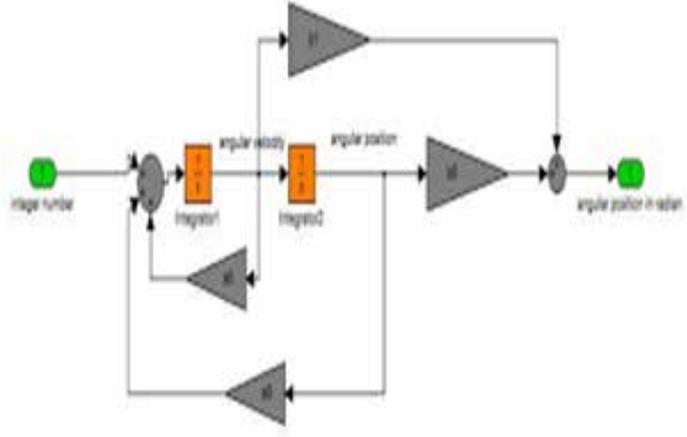


Figure 2: Second order system canonical form (Alexander, 2013)

Comparing the tricopter transfer function given in equation 14 with the general form of 2nd order

$$G(s) = \frac{b_1 * s + b_0}{s^2 + a_1 * s + a_0} = \frac{k}{s^2} \quad \dots\dots (34)$$

where, K represents K_ϕ, K_θ, K_ψ .

The study assumed coefficients, b_0 equals to K_0 and b_1, a_1, a_0 equals 0. Therefore, the general state space form becomes

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ K_\theta \end{bmatrix}, \quad C = [1 \quad 0].$$

The structural diagram of the pitch angle model is as shown in figure 3,

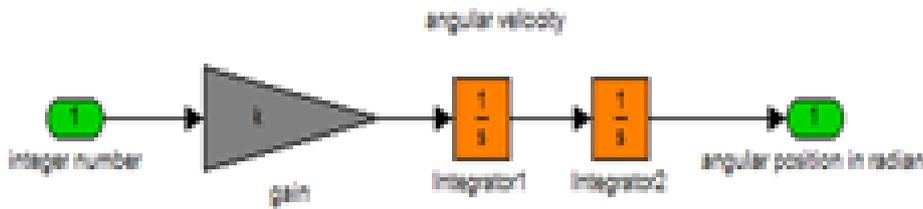


Figure 3: Diagram of the pitch angle model (Alexander, 2013)

Acquiring the desired characteristic poles using the Ackerman formula (i.e. Equation (24)), the matrix polynomial becomes,

$$\alpha_c(s) = (s + \mu_1)(s + \mu_2) = s^2 + (\mu_1 + \mu_2)s + \mu_1\mu_2 = 0, \quad \dots\dots\dots(36)$$

Taking into account that the desired poles $s_{1,2} = -3.5367 \pm j3.5363$, and substituting into equation (36), the matrix polynomial becomes

$$\alpha_c(s) = s^2 + 7.0734s + 25 = 0, \quad \dots\dots\dots(37)$$

Combining equations (37) and (35), the matrix polynomial becomes

$$\begin{aligned} \alpha_c(A) &= A^2 + 7.0734 * A + 25 * I = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 7.0734 * \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 65 * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 7.0734 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} \\ &= \begin{bmatrix} 25 & 7.0734 \\ 0 & 25 \end{bmatrix} \quad \dots\dots\dots(38) \end{aligned}$$

To obtain the matrix K , recall the parameters in equation (22)

$$[B \quad A * B]^{-1}$$

$$A * B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 \\ K \end{bmatrix} = \begin{bmatrix} K \\ 0 \end{bmatrix},$$

thus, $[B \quad A * B]^{-1} = \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix}$

The matrix K can be obtained as:

$$\begin{aligned} K &= [0 \quad 1] * [B \quad A * B]^{-1} * \alpha_c(A) = [0 \quad 1] * \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} * \begin{bmatrix} 25 & 7.0734 \\ 0 & 25 \end{bmatrix} \\ &= [0 \quad 1] * \begin{bmatrix} 0 & 25 * K \\ 25 * K & 7.0734 * K \end{bmatrix} = [25 * k \quad 7.0734 * K] \end{aligned}$$

therefore $K = [K_1 \quad K_2] = [25 * K \quad 7.0734 * K], \quad \dots\dots\dots(39)$

The closed-loop system matrix A_{cl} is mathematically represented as

$$A_{cl} = \begin{bmatrix} 0 & 1 \\ -a_0 - K_1 & -a_1 - K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 * K & -7.0734 * K \end{bmatrix}$$

The state-space equations for closed-loop system with Simulink model is shown in Figure 4,
 $\dot{x}(t) = A_{cl} * x(t) + B * u(t)$ (40)

$$y(t) = C * x(t)$$

where, $A_{cl} = \begin{bmatrix} 0 & 1 \\ -25 * K & -7.0734 * K \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [1 \ 0]$.

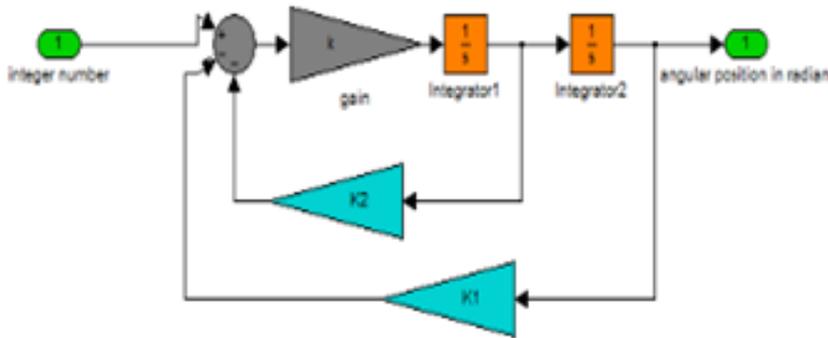


Figure 4 Closed loop system with desired poles(Alexander, 2013)

The system keeps its position at zero when applying regulators. However, the attitude controllers obey desired values of the pitch, roll and yaw that are inputted. The output of the closed-loop system equals to the input; if the gain of the closed loop transfer function equals to one. The transfer function of the system

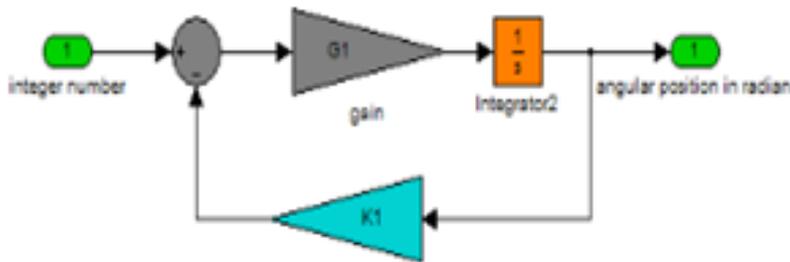


Figure 5 Feedback systems with desired poles for finding the transfer function(Alexander, 2013)

The inner transfer function is given as

$$G_f(s) = \frac{\frac{k}{s}}{1 + \frac{k}{s} + K_2} = \frac{k}{s + k * K_2}$$
(41)

The transfer function of the regulator G(s) obtained from equation 41 is given as:

$$G(s) = \frac{\frac{k}{s + k * k_2} * \frac{1}{s}}{1 + \frac{k}{s + k * k_2} * \frac{1}{s} * k_1} = \frac{k}{s^2 + k * k_2 * s + k * k_1}$$
(42)

The gain of the regulator G(s) can be derived as

$$G(0) = \frac{1}{k_1} \tag{43}$$

From equation (43), it can be implied that G(s) should be multiplied by k₁ for a unity gain. The controller final transfer function can be deduced as

$$G(s) = \frac{k * k_1}{s^2 + k * k_2 * s + k * k_1} \tag{44}$$

The controller was implemented in MATLAB Simulink environment using equation (14). The roll control, pitch control and yaw control blocks are based on the coefficients given in equation (39).

Implementation of Controller

The study considered position control of two parts; height control along the Z axis and control along the XY plane (2D). The height control is based on the linearized model (equation 17) and the Ackerman approach

where $k = \frac{k_T}{m}$. From linearized equation, the required acceleration can be deduced as written in equation 45.

$$\begin{aligned} \phi^d(s) &= g * (s^2 r_x(s) * \sin\psi(s) - s^2 r_y(s) * \cos\psi(s)) \\ \theta^d(s) &= g * (s^2 r_x(s) * \cos\psi(s) + s^2 r_y(s) * \sin\psi(s)) \end{aligned} \tag{45}$$

where, g is gravitational acceleration, $\phi^d(s)$ and $\theta^d(s)$ are X and Y desired values.

The control law based on equation (45) was implemented in MATLAB Simulink environment as shown in figure 6.

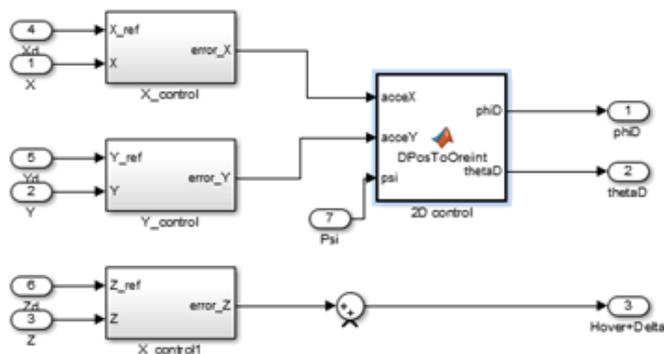


Figure 6: Altitude control implementation on Simulink

For a constant flight where the yaw angle is zero, the controller is simplified and represented by

$$\begin{aligned} \phi^d(s) &= -g * s^2 r_y(s) * \cos\psi(s) \\ \theta^d(s) &= g * s^2 r_x(s) * \cos\psi(s) \end{aligned} \tag{46}$$

This revealed that controller X control, Y control and Z control have the same structure as the attitude control. The coefficients of Z control are obtained from Ackerman approach while those of X and Y was manually adjusted.

The response of the input control can be observed in Figures 7 to 12. The result shows Altitude, Roll, Pitch and Yaw with respective angles.

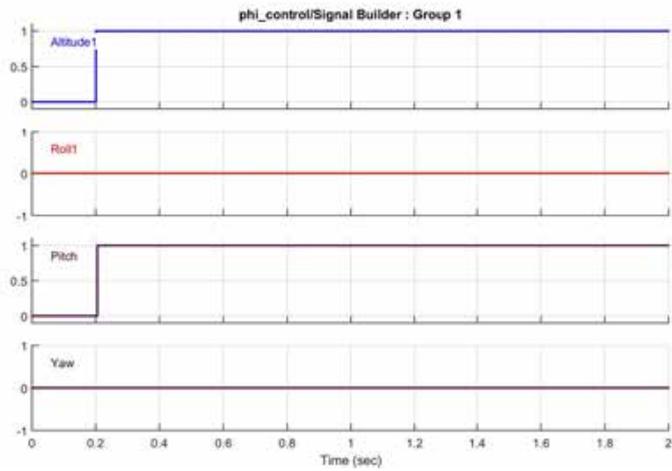


Figure 7: Input controls to the Tricopter model

The tricopter was analyzed with input signals shown in Figure 7. The altitude input and pitch input were step commands since upward motion is similar to a step input with an initial value of 0 to a final value of 1. The roll and yaw inputs are constant input signals of zero. The output response for the pitch and yaw angles are observed in Figures 8 and 9.

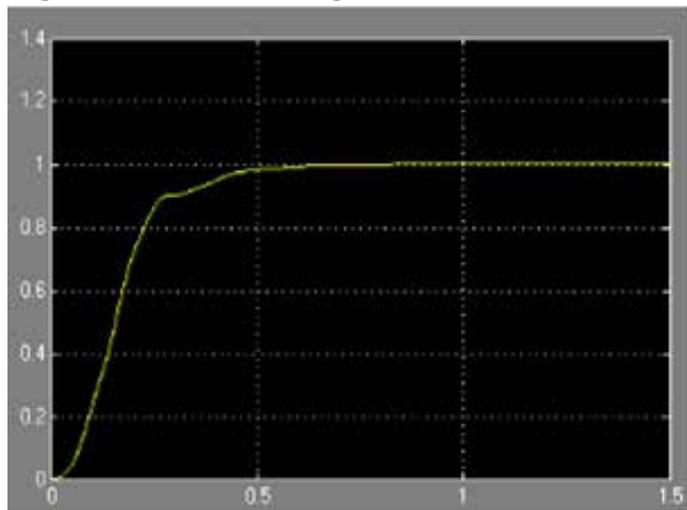


Figure 8: Step response of the pitch angle with a controller

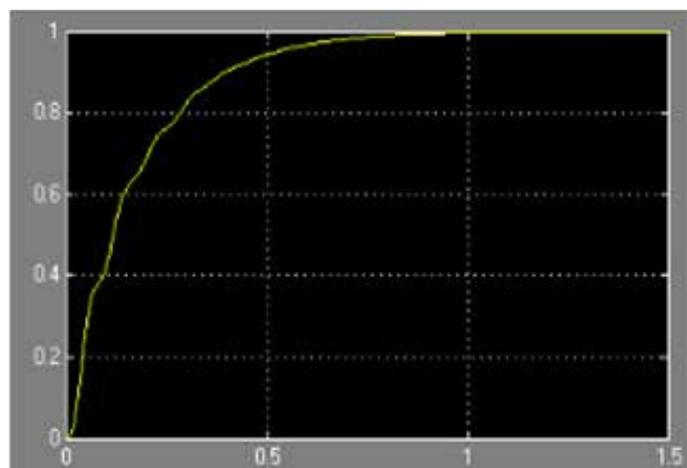


Figure 9: Step response of yaw angle with a controller

It can be seen that the controllers provide quality requirements as T_s equals 0.8s and percentage overshoot $\%M_p$ equals 4.32%. The steady state error is almost zero. It can be observed that the medication delivery tricopter response is made stable with the aid of a controller. The result shows that the tricopter requires a controller for its stability.

The altitude controller results revealed the step responses observed in Figures 10-12.

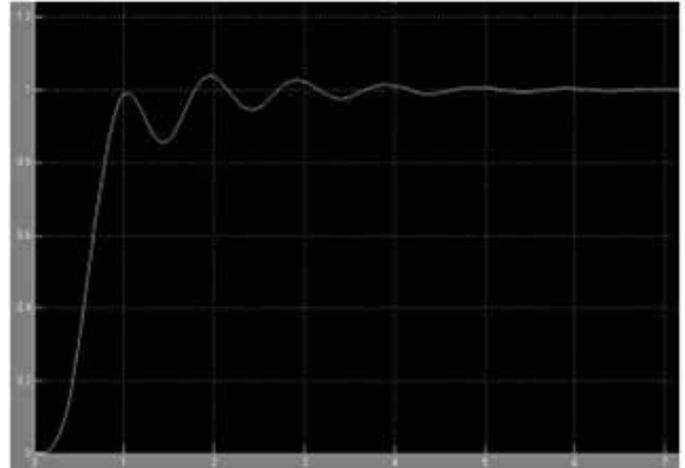


Figure 10: The step response of the tricopter X position

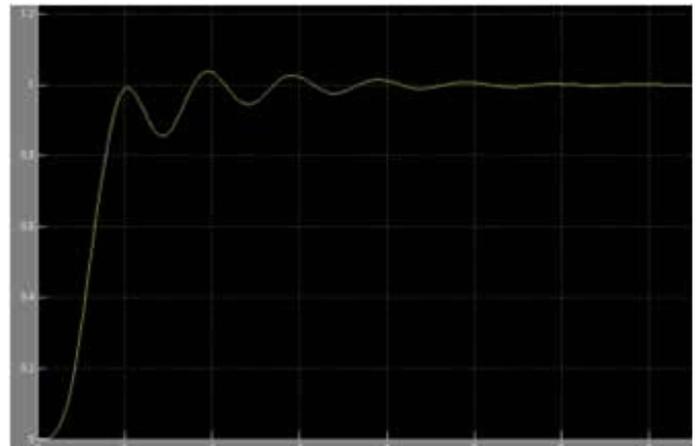


Figure 11: The step response of the tricopter Y position

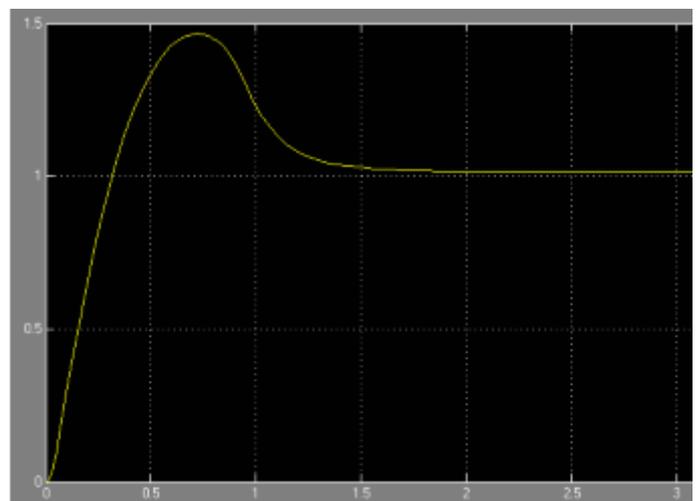


Figure 12: The step response of the Tricopter Z position

There are no overshoots along X and Y axes and settling time is at 3s. The overshoot along Z axis is about 50% and settling time is about 2s.

CONCLUSION

The goal of the study was to design a controller for a medication delivery tricopter. Aerodynamic principles are based on dynamic laws which guide the mechanics of air vehicles. The study adopted control law using Ackerman approach due to its simplicity and robustness. Ackerman method is a pole placement method of designing a controller. This entails desired system parameters such as settling time, maximum overshoot, and damping factors. The research deployed damping factor to obtain the desired poles for closed loop control system. Desired poles are

utilized to design a controller suitable for the system and to satisfy predefined conditions or parameters. Furthermore, the tricopter model was implemented in the MATLAB/Simulink environment. The model was tested and was found to be stable and controllable.

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