



THE HARMONIC RESPONSE OF EULER-BERNOULLI BEAM SUBJECTED TO MOVING FORCE

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ABSTRACT

This paper investigates the dynamic response of Euler-Bernoulli Beam subjected to Harmonic moving force. The governing partial differential equation of order four was reduced to a second order ordinary differential equation by normalizing the governing equation. The reduced second order equation was solved using finite difference method. The deflection for various parameters of the beam was considered for both damped and undamped beam. It was observed that there was an increase in the response amplitude of the beam as the constant speed of the beam, amplitude of the harmonic force increases but decreases as circular frequency of the harmonic force increases. It was also found that the response amplitude of the undamped beam is greater than the response amplitude of the damped beam. Hence, the damping effect is of significant importance when considering the deflection of the beam subjected to harmonic moving force.

Keywords: *Euler-Bernoulli Beam, Moving Force, Harmonic, Amplitude.*

INTRODUCTION

Vibration is a mechanical phenomenon whereby oscillations (periodic or random) occur about an equilibrium point. It is the motion of a particle or a body system of connected bodies displaced from a position of equilibrium.

When an applied load varies as a sine or a cosine function, it is called harmonic loading.

Sine loading:

$$f(t) = F \sin(\omega Ft)$$

Cosine loading:

$$f(t) = F \cos(\omega Ft)$$

The applied force or displacement excitation may be harmonic, nonharmonic but periodic, nonperiodic, or random in nature. The response of a system to a harmonic excitation (loading) is called harmonic response.

The nonperiodic excitation may have a long or short duration. The response of a dynamic system to suddenly applied nonperiodic excitations is called transient response. If the frequency of excitation coincides with the natural frequency of the system, the response of the system will be very large. This condition is known as resonance, which should be avoided to prevent failure of the system.

The examples of harmonic motion include the vibration produced by an unbalanced rotating machine, the oscillations of a tall chimney due to vortex shedding in a steady wind, and the vertical motion of an automobile on a sinusoidal road surface. The dynamic effects of a load on beam and beam-like structural members play significant role in railway tracks, road tracks and highway pavement designs. In modern engineering practices, beamlike structures resting on both variable and constant elastic foundation have wide applications and for this reason several authors have investigated the dynamic deflection of beam.

When the Chester rail bridge collapsed in England, various kinds of moving load problem associated with structural dynamics have been presented in excellent monograph by Fryba (1977). In the Fryba book detailed solution of the problem of a constant force moving along infinite beam over an elastic foundation including its all possible speed and values of viscous damping is presented. Dynamic problem of a simply supported beam subjected to a constant force moving at a constant speed is analyzed by Olsson (1991). Analytical and finite element solutions to this fundamental moving load problem is shown and the result given by the author and other investigators are intended to give a basic understanding of the moving load problem and some computational

algorithms discussed. Furthermore, Kenny (1954), took up the problem of investigating the dynamic response of infinite elastic beam on elastic foundation when the beam is under the influence of a dynamic load moving with constant speed. He included the effect of viscous damping in the governing differential equation of motion. Eisenberger and Clastornik (1987) solved the problem of a beam on a parametric elastic foundation and presented a finite element procedure for analyzing the flexural vibrations. Cao and Zhong (2008) solved the problem of a beam on a Pasternak foundation and under a moving load. The method of Fourier integral is used to obtain the solution to the formulation of the problem. The effect of the moving load velocity on the dynamic displacement response of the beam is discussed. Also, Awodola (2005) considered the influence of foundation and axial force on the vibration of thin beam under variable harmonic moving load. The technique is based on the finite Fourier sine transformation. More recently, Oni and Awodola (2010) investigated the dynamic behaviour under moving concentrated masses of simply supported rectangular plates resting on variable Winkler elastic foundation.

Arash *et al.*, (2002) analyzed the dynamic response of Timonshenko beams under moving mass using numerical method called Discrete Element Technique (DET). Gbadeyan and Idowu (2002) studied the dynamic behaviour of a prestressed Bernoulli beam carrying an added mass at one end, and arbitrarily supported at the other end which is under a number of moving concentrated masses. Approximate analytical technique was employed to solve the equation of motion of the beam which involves taking the generalized finite integral transform of the initial-boundary value problem and then solving the resulting ordinary differential equation with the initial conditions using modified strubbles technique. Method of determining the dynamic response of prismatic damped Euler-Bernoulli beam subjected to distributed and concentrated load was presented by Abu-Hilal (2003). He also presented the green functions for various beam with homogeneous and elastic boundary condition. The vibrational analysis of Euler-Bernoulli beam with viscous damping coefficient subjected to uniform partially distributed moving load was investigated by Gbadeyan and Usman (2003). It was observed from the result that the damping increased with an increase in the amplitude of the deflection of constant fixed length of the beam. Akinpelu (2012), examined the response of viscously Bernoulli beam to uniform partially distributed moving loads. The author observed that the beam has more

than one mode of vibration with each mode having a different natural frequency. It was discovered that as the mass of the load increases the amplitude is also increases and the value of the magnification factor occurs for a value ω less than one for $\epsilon = 0.1$. The dynamics responses of a beam acted upon by moving masses have been studied extensively in connection with the design of railway tracks and machining processes by Lee (1996). The equation of motion in matrix form has been formulated for the dynamics response of a beam acted upon by a moving mass by using Lagrangian approach and the assumed mode method, and found that separation of the mass from the beam may occur for a relatively slow speed and small mass when the beam is clamped at both ends. The dynamic analysis of prebeam was considered by Adetunde (2007). The finite difference method was used to solve the pertinent initial boundary value problem numerically and he concluded that the deflection of the moving mass pre-Bernoulli beam was greater than those of the moving force. Dynamic response of loads on viscously damped axial force Rayleigh beam was studied by Adetunde and Baba Seidu (2008). The theory is based on orthogonal functions and the results indicates that the governing differential equation can be transformed into a series of couple ordinary differential equations which is the solution for the corresponding moving distributed force. The resulting governing differential equation is solved numerically by finite central difference method where he concluded that the deflection due to moving mass is greater than that due to moving force. The response of initially stressed

Euler with an attached mass to uniform partially distributed moving loads was carried out by Adetunde *et al.*, (2007). The resulting coupled partial differential equation is solved using finite difference method. It was found that the response amplitude increases as mass of the load increases under a moving force problem and also that the response amplitude increases with an increase in the mass of the load for various values of time t and. Axial loaded beams on elastic foundation under moving harmonic loads was investigated by Kim Seong The work was based on the vibration and stability of an infinite Euler-Bernoulli beam on a Winkler foundation by applying a static axial force and a moving load with either constant or harmonic amplitude variations to excite the system and some important results were obtained from the process of changing relative parameters. Response of beam on visco moving distributed load was examined by Roman and Wlodzimierz (2008) where the dynamical problems caused by a distributed load

which was acting on the beam. Jaiswal and Iyengar (1993) examined dynamic response of a beam on elastic foundation of finite depth under a moving force. The dynamics of the infinite beam on a finite elastic foundation base subjected to a moving load was studied. Again, the effects of various parameters such as foundation mass, velocity of the moving load, damping and axial force on the beam were investigated.

MATERIALS AND METHODS

The governing equation

Vibration of Euler-Bernoulli beam subjected to harmonic moving force has the governing equation of the form

$$EI \frac{\partial^4 V(x,t)}{\partial t^4} + \mu A \frac{\partial^2 V(x,t)}{\partial t^2} + 2\mu w_b \frac{\partial V(x,t)}{\partial t} = f(x,t) \quad \dots\dots\dots 1$$

with

$$f(x,t) = \delta(x-ct)P(t) \quad \dots\dots\dots 2$$

x is the spatial coordinate

t is the time

$\frac{\partial^n}{\partial x^n}$ is the n th partial derivative with respect to x

$V(x,t)$ is the deflection of the beam

E is the Young's modulus

I is the moment of inertia of the beam's cross section about the neutral axis

μA is the Mass per unit length of the beam

w_b is the Damping coefficient

Q is the Amplitude of the force

Ω is the Circular frequency of the harmonic force

$\delta(\cdot)$ is the Dirac-delta function such that

$$\delta(x-ct) = \begin{cases} 0 & \text{for } x = 0 \\ \infty & \text{for } x \neq 0 \end{cases} \quad \dots\dots\dots 3$$

With the property that

$$\int_a^b \delta(x-ct)f(x) = \begin{cases} f(vt) & a < ct < b \\ 0 & a < b < vt \\ 0 & vt < a < b \end{cases} \quad \dots\dots\dots 4$$

Hence, the governing equation becomes

$$EI \frac{\partial^4 V(x,t)}{\partial t^4} + \mu A \frac{\partial^2 V(x,t)}{\partial t^2} + 2\mu w_b \frac{\partial V(x,t)}{\partial t} = \delta(x-ct)P(t) \quad \dots\dots\dots 5$$

Where $P(t) = Q\sin\Omega t$ is the time variable concentrated harmonic force With the boundary conditions

$$V(0,t) = 0 = V(l,t) \quad \dots\dots\dots 6$$

$$\frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(l,t)}{\partial x^2} \quad \dots\dots\dots 7$$

Without loss of generality, one can consider the initial conditions of the form

$$V(x, 0) = 0 = \frac{\partial V(x, 0)}{\partial t} \dots\dots\dots 8$$

Method of Solution

Assume a solution such that the transverse vibration of the beam may be expressed in the following series form

$$V(x, t) = \sum_{m=1}^{\infty} X_m(x)T_m(t) \dots\dots\dots 9$$

Substituting (9) into (5), we have

$$EI \sum_{m=1}^{\infty} X_m^{iv}(x)T_m(t) + \mu A \sum_{m=1}^{\infty} X_m(x)T_m''(t) + 2\mu w_b \sum_{m=1}^{\infty} X_m(x)T_m'(t) = \delta(x - ct)Q \sin \Omega t \dots\dots\dots 10$$

Furthermore, the forcing term f(x,t) defined in equation (2) can also be expressed as

$$f(x, t) = \sum_{m=1}^{\infty} T_{fi}(t)X_m(x) \dots\dots\dots 11$$

Substituting (11) into (2), we have

$$\sum_{m=1}^{\infty} T_{fi}(t)X_m(x) = \delta(x - ct)Q \sin(\Omega t)X_m(x) \dots\dots\dots 12$$

$$T_{fi}(t)X_m(x)X_n(x) = X_n(x)X_m(x)\delta(x - ct)Q \sin(\Omega t) \dots\dots\dots 13$$

Integrating both sides of (13) with respect to x along the length L of the beam, we have

$$\int_0^L T_{fi}(t)X_m(x)X_n(x)dx = \int_0^L X_n(x)X_m(x)\delta(x - ct)Q \sin \Omega t dx \dots\dots\dots 14$$

Using condition of equation (4), (14) can be rewritten as

$$\int_0^L T_{fi}(t)X_m(x)X_n(x)dx = X_m(x)X_n(ct)Q \sin \Omega t dx \dots\dots\dots 15$$

For the boundary conditions given under the governing equation

$$X_m(x) = \sin \left[\frac{m\pi x}{L} \right] \dots\dots\dots \dots\dots 16$$

To obtain a set of exact governing differential equation for the simply supported beam under consideration, we substitute (16) into (14) to obtain

$$\sum_{m=1}^{\infty} T(t) \sin \left[\frac{n\pi}{L} x \right] dx = Q \sin \Omega t \int_0^L \delta(x - ct) \sin \frac{n\pi}{L} x dx \sin \frac{m\pi}{L} x dx \dots\dots\dots 17$$

Evaluating the above integral, we have

$$\int_0^L \delta(x - ct) \sin \frac{n\pi}{L} x dx = \sin \left[\frac{n\pi}{L} ct \right] \sin \left[\frac{m\pi}{L} x \right] \dots\dots\dots 18$$

Hence equation (17) can be written as

$$\sum_{n=1}^{\infty} T(t) \sin \left[\frac{n\pi}{L} x \right] dx = Q \sin \Omega t \sin \left[\frac{n\pi}{L} ct \right] \dots\dots\dots 19$$

By replacing the right-hand side of (19) with the right-hand side of (10), we finally obtain

$$EI \frac{m^4 \pi^4}{L^4} \sin \frac{m\pi x}{L} x T_m(t) + \mu A \sin \frac{m\pi}{L} x T_m''(t) + 2\mu w_b \sin \frac{m\pi x}{L} x T_m'(t) = Q \sin \Omega t \sin \left[\frac{n\pi}{L} ct \right] \sin \left[\frac{m\pi}{L} x \right] \dots\dots\dots 20$$

which can be re-arranged as

$$\begin{aligned} & \mu A \sin \frac{m\pi}{L} x T_m''(t) + 2\mu w_b \sin \frac{m\pi x}{L} x T_m'(t) + EI \frac{m^4 \pi^4}{L^4} \sin \frac{m\pi x}{L} x T_m(t) \\ & = Q \sin \Omega t \sin \frac{n\pi c}{L} t \sin \frac{m\pi}{L} x \end{aligned} \dots\dots\dots 21$$

Dividing through by the coefficient of T'' ,

$$T_m''(t) + \frac{2\mu w_b \sin \frac{i\pi x}{L} x}{\mu A \sin \frac{m\pi}{L} x} T_m'(t) + \frac{EI \frac{m^4 \pi^4}{L^4} \sin \frac{m\pi x}{L} x}{\mu A \sin \frac{m\pi}{L} x} T_m(t) = \frac{Q}{\mu A \sin \frac{m\pi}{L} x} \sin \Omega t \sin \frac{n\pi c}{L} t \sin \frac{m\pi}{L} x \dots\dots\dots 22$$

With further simplification, we have

$$T_m''(t) + 2w_b T_m'(t) + EI \frac{m^4 \pi^4}{\mu AL^4} T_m(t) = \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t \dots\dots\dots 23$$

In order to solve the above second order ordinary differential equation, we shall employ finite central difference method, so that

$$\begin{aligned} T_m''(t) &= \frac{(T_{m+1} - 2T_m + T_{m-1}))}{h^2} \\ T_m'(t) &= \frac{(T_{m+1} - T_{m-1}))}{2h} \end{aligned} \dots\dots\dots (24)$$

$$\frac{(T_{m+1} - 2T_m + T_{m-1}))}{h^2} + 2w_b \left(\frac{(T_{m+1} - T_{m-1}))}{2h} \right) \dots\dots\dots 25$$

$$+ EI \frac{m^4 \pi^4}{\mu AL^4} T_m = \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t$$

$$\frac{T_{m+1} - 2T_m}{h^2} = \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t - \frac{T_{m-1}}{h^2} \dots\dots\dots 26$$

$$- 2w_b \left(\frac{(T_{m+1} - T_{m-1}))}{2h} \right) - EI \frac{m^4 \pi^4}{\mu AL^4} T_m$$

$$\begin{aligned} \left(\frac{1 + hw_b}{h^2} \right) T_{m+1} &= \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t \\ - \left(\frac{2 + h^2}{h^2} \left(EI \frac{m^4 \pi^4}{\mu AL^4} \right) \right) T_m &- \left(\frac{1 + hw_b}{h^2} \right) T_{m-1} \end{aligned} \dots\dots\dots 27$$

Multiply through by h²

$$\begin{aligned} (1 + hw_b) T_{m+1} &= h^2 \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t \\ - \left(2 + h^2 \right) EI \frac{m^4 \pi^4}{\mu AL^4} T_m &- (1 + hw_b) T_{m-1} \end{aligned} \dots\dots\dots 28$$

so that

$$T_{m+1} = \frac{h^2}{(1 + hw_b)} \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t - \left(\frac{\left(2 + h^2 \left(EI \frac{m^4 \pi^4}{\mu AL^4} \right) \right)}{(1 + hw_b)} \right) T_m - \frac{(1 + hw_b)}{(1 + hw_b)} T_{m-1} \dots\dots\dots 29$$

Furthermore,

$$T_{m+1} = \frac{h^2}{(1 + hw_b)} \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t - \left(\frac{\left(2 + h^2 \left(EI \frac{m^4 \pi^4}{\mu AL^4} \right) \right)}{(1 + hw_b)} \right) T_m(t) - T_{m-1} \dots\dots\dots 30$$

Finally, from equation (9) we have that

$$V(x,t) = \sum_{m=1}^{\infty} \left(\frac{h^2}{(1+hw_b)} \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t - \left(\frac{2 + h^2 \left(\frac{EI m^4 \pi^4}{\mu AL^4} \right)}{(1+hw_b)} \right) T_m(t) - T_{m-1} \right) \sin \frac{m\pi}{L} x \dots\dots\dots 31$$

Equation (31) is the deflection of damped Euler-Bernoulli Beam subjected to harmonic moving force.

For the undamped Euler-Bernoulli Beam

Here, since $w_b = 0$, equation (23) becomes

$$T_m''(t) + EI \frac{m^4 \pi^4}{\mu AL^4} T_m(t) = \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t \dots\dots 32$$

Applying finite central difference method to equation (32) and making T_{m+1} the subject of the formula or equating $w_b = 0$ in equation (31), the deflection of Undamped Euler-Bernoulli Beam subjected to harmonic moving force is

$$V(x,t) = \sum_{m=1}^{\infty} \left(h^2 \frac{Q}{\mu A} \sin \Omega t \sin \frac{n\pi c}{L} t - \left(2 + h^2 \left(\frac{EI m^4 \pi^4}{\mu AL^4} \right) \right) T_m(t) - T_{m-1} \right) \sin \frac{m\pi}{L} x \dots\dots\dots 33$$

RESULTS AND DISCUSSION

Beam dimension and specification:

Length(L)=10 m

Density of the mass (ρ) = 164000000kg/m³

Mass per unit length of the beam $\rho A=0.98$

Rigidity of the Beam $EI = 1.74 \times 10^{-5}m^4$

Amplitude of the force $Q=2.4,2.7,3.0kN$

Circular Frequency of the force $\Omega=5,10,20rad/s$

Constant Speed $c=5,7,10m/s$

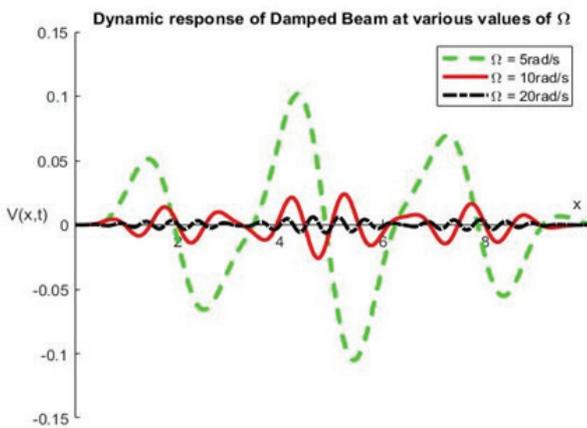


Figure (1.0): Dynamic response of damped beam at various values of Ω

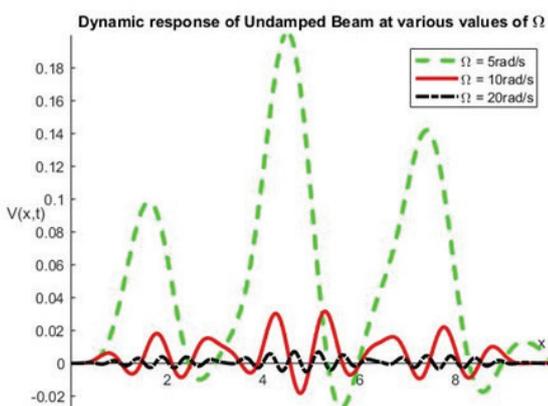


Figure (1.1): Dynamic response of undamped beam at various values of Ω

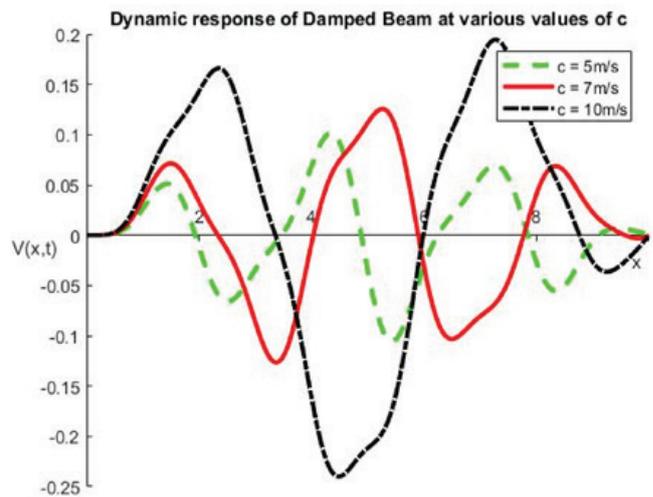


Figure (1.2): Dynamic response of damped beam at various values of c

Figure 1. Displays the shows the dynamic response of the damped beam at various values of circular frequency of the harmonic force. It is observed that there is decrease in the response amplitude of the deflection of the beam at the positive axis but increases along the negative axis as the circular frequency increases. It is also found that the number of amplitude increases as the circular frequency increases.

Figure 2 Shows the dynamic response of the undamped beam at various values of circular frequency of the harmonic force. It is observed that there is a decrease in the response amplitude of the deflection of the beam as the circular frequency increases.

Comparing Figure 1 and 2, we could see that Figure 1 is more negatively deflection that the undamped beam in Figure 2 at 5rad/s. It is also seen that at 20rad/s, the deflection are the same for damped and undamped beam.

Figure 3 shows the dynamic response of the damped beam at various values of constant speed. It is found that the response amplitude of the beam increases along the positive axis but decreases along the negative axis as the speed increases. It is also observed that the number of amplitude decreases as the speed increases.

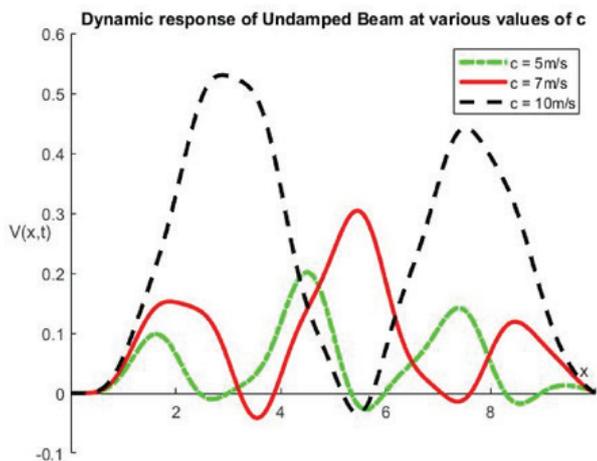


Figure (1.4): Dynamic response of damped beam at various values of c

Figure 5 shows the dynamic response of the undamped beam at various values of the amplitude of the force. It is observed that the deflection of the beam increases in peak amplitude as the amplitude of the harmonic force increases along the positive axis but decreases along the negative axis.

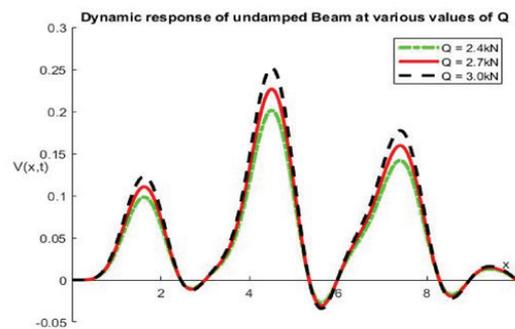


Figure (1.5): Dynamic response of damped beam at various values of Q

Figure 6 shows the dynamic response of the undamped beam at various values of the amplitude of the force. It is observed that the deflection of the beam increases in peak amplitude as the amplitude of the harmonic force increases.

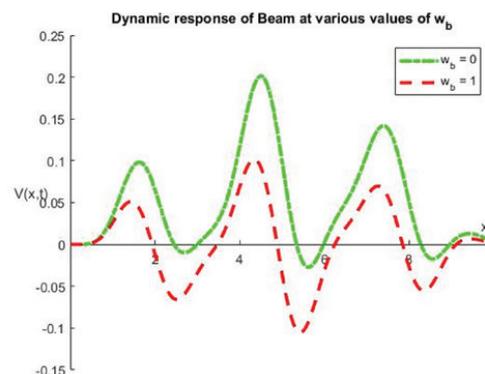


Figure (1.6): Dynamic response of beam at various values of w_b

Figure (1.6) Shows the dynamic response of the beam at various values of the damping coefficient. It is found that the response amplitude of the beam decreases as the damping coefficient increases.

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