



ANALYSIS OF A NONLINEAR HEAT CONDUCTION MODEL OF THE HUMAN HEAD.

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ABSTRACT

This research work is an account of the numerical analysis of a steady, one dimensional, spherically symmetric heat conduction equation modeling a simplified human head under the influence of environmental temperature variations. The governing energy balance equation together with the corresponding boundary conditions was transformed into dimensionless boundary value problem. The boundary value problem was solved numerically using fourth order Runge-kutta method with shooting technique. The results from the temperature profile revealed that the temperature of the human head is higher at the core and decreases transversely with minimum value towards the periphery. The effects of parameters like Biot number due to an increase in the rate of heat transfer, thermogenesis heat production and metabolic thermogenesis slope production on the temperature of the human head are examined. The results of the study proved that there is a perfect agreement in the core temperature values of the present work using various parameter values when compared with the reports of previous findings in the literature.

Keywords: *Heat conduction, Human Head, Periphery, Steady, Thermogenesis*

INTRODUCTION

The human head is the part of the body containing the brain and the organs of special senses like the mouth, nose, ears and related structures. The brain is the main part of the central nervous system including the spinal cord that controls all the body processes (Assefa and Tsige, 2002). Heat transfer systems contain many sources of nonlinearity including temperature dependent material properties, radiation boundary conditions and internal source terms. One remarkable features of the human thermoregulatory system is that a core temperature near 37°C over a wide range of environmental conditions and during thermal stress, such as exposure to radio frequency waves, can be maintained (Assefa and Tsige, 2002). The amount of blood flow to the body varies over a wide range depending upon the need for its functions. In heat transfer for systematic thermoregulation, blood is known to have a dual influence on the thermal energy balance either by heat source or sink depending on the local temperature (Duggan and Goodman, 1986).

Temperature is one of the most critical factor in determining not only the biochemical and metabolic processing rates but also brain-to-body weight ratios. In all, core brain temperature is higher than core body temperature; however, these two temperatures correlate very well. Heat dissipation in the head is necessary due to continuous heat production and tight control occurring via radiation, conduction, convection, and evaporation. The human head is the home to all the body's major sensory organs most especially the brain. It receives information from different parts of the body, interprets the information and guides the body in its response.

In a research work from Anderson and Arthurs, 1981, they derived the complementary variation principles and the associated approximate solutions for a nonlinear model of heat conduction in the human head. In their work, human head temperature is higher at the core and decreases transversely with minimum value at the periphery.

Finite difference numerical solution was presented by Celik and Gokman, 2003 to solve a nonlinear model for the distribution of temperature in the human head with variable thermal conductivity. Their results showed that the temperature of the human head increases as the thermogenesis heat parameter increases with decrease in the biot number. Colin *et al*, 1966 gave the role of mathematical models in the treatment and diagnosis of the temperature distribution of the human head, predicting the time course of temperature where invasive thermometry is lacking. Duggan and Goodman, 1986 applied the theory of maximum principle to a non-linear heat conduction model of the human head to obtain accurate analytical upper and lower bounding curves and then concluded that increase in biot number decreases the temperature of the human head. Fenuga *et al*, 2013 considered the effects of thermogenesis

parameters and Biot numbers on a nonlinear heat conduction model of the human head. Their result is in agreement with Celik and Gokman, 2003. Flesch, 1975 summarized the experimental data of Colin *et al.*, 1966 and interpreted it successfully on a theoretical basis. He concluded that generation of heat is greater in the periphery of the human head than in the center and sensitive to ambient temperature changes at the periphery. Froese and Burton, 1957 provided the detailed analysis on heat loss from the human head. Gorbach *et al.*, 2003 gave the intraoperative infrared functional imaging of the human brain. Gray, 1980 investigated a simplified linear model for heat source distribution in the human head and showed that in response to the drop in ambient temperature, the peripheral heat generation increases more than the central heat generation. Kwon *et al.*, 2011 explained how GSM mobile phone radiation suppresses brain glucose metabolism. Makinde, 2010 provided a non-perturbative, semi conductive solution to a non linear singular boundary value analytical problem modeling the distribution of heat sources in the human head and their dependence on environmental temperature by using Adomian decomposition method. His results show that human temperature is higher at the core and decreases transversely with minimum value at the periphery. Nakagawa *et al*, 2011 gave the effects of decompressed hemicraniectomy on brain temperature after severe brain injury. Nybo and Nielsen, 2011 gave how middle cerebral artery blood velocity in human is reduced with hyperthermia during prolonged exercise. Ricquier, 2006 provided the fundamental mechanism of thermogenesis. Stone *et al*, 1997 in a work developed intraoperative measurement of human brain temperature.

This work is focused on temperature dependent heat generation within the human head.

It extends the work of Makinde, 2010 by using Runge-Kutta Method with Shooting technique different from Adomian decomposition Method used by Makinde, 2010 to analyse a nonlinear heat conduction model of the human head.

MATERIALS AND METHOD

We consider a steady, one-dimensional spherically symmetric, heat conduction equation modeling a simplified human head under the influence of environmental temperature variation. Following Anderson and Arthurs (1981), Duggan and Goodman (1986) together with Makinde, 2010, the governing energy balance equation is:

$$\frac{d^2T}{d\bar{r}^2} + \frac{2}{\bar{r}} \frac{dT}{d\bar{r}} + \frac{\delta}{\kappa} e^{-\alpha T} = 0 \quad \dots \quad (1)$$

and its corresponding boundary conditions are:

$$\frac{dT}{d\bar{r}} = 0 \quad \text{at} \quad \bar{r} = 0 \quad \dots \quad (2)$$

$$-\kappa \frac{dT}{d\bar{r}} = \beta(T - T_a) \text{ at } \bar{r} = R \quad \dots \quad (3)$$

where κ is the average thermal conductivity inside the head, T is the absolute temperature, $0 \leq \bar{r} \leq R$ is the radial distance measured from the centre to the periphery of the head, β is the heat exchange coefficient from the head to the surrounding medium, T_a is the ambient temperature, α is the metabolic thermogenesis slope parameter, δ is the thermogenesis heat production parameter and \bar{r} is the radial distance measured from the center to the periphery of the head.

Introducing the dimensionless variables

$$\theta = \frac{T}{T_a} \Rightarrow T = T_a \theta \quad \dots \quad (4)$$

$$\tau = \frac{\bar{r}}{R} \Rightarrow \bar{r} = R\tau \quad \dots \quad (5)$$

Substituting (4) and (5) in equations (1),(2) and (3) gives the equation

$$\frac{d^2\theta}{d\tau^2} + \frac{2}{\tau} \frac{d\theta}{d\tau} + \lambda e^{-m\theta} = 0 \quad (6)$$

where $\lambda = \frac{\delta R^2}{T_a \kappa}$, $m = \alpha T_a$ and $B_i = \frac{\beta R}{\kappa}$

with the boundary conditions

$$\frac{d\theta}{d\tau} = 0 \text{ at } \tau = 0 \quad (7)$$

$$\frac{d\theta}{d\tau} = B_i(1 - \theta) \text{ at } \tau = 1 \quad (8)$$

Where θ is the variable representing head temperature, B_i is the Biot number, m is the metabolic thermogenesis slope parameter and λ is the thermogenesis heat production parameter. Then, using the Runge-Kutta numerical Method with Shooting Technique, the following tabular and graphical results were obtained.

RESULTS AND DISCUSSION

This section presents the numerical and graphical results of the human head temperature distribution.

Temperature distribution or profiles within the human head are highly influenced by the geometry and the rate of metabolic heat production parameter.

Table 1. Comparison showing the core temperature at various parameter values using Adomian Decomposition Method (ADM) and Runge-Kutta Method (RKM).

B_i	λ	m	Makinde (2010) ADM $\theta(0)$	Present work (2021) RKM (Shooting technique) $\theta(0)$
0.1	1.0	1.0	1.67070777	1.67070777
0.5	1.0	1.0	1.24606780	1.24606780
1.0	1.0	1.0	1.16081981	1.16081981
1.0	3.0	1.0	1.39599464	1.39599465
1.0	5.0	1.0	1.56987889	1.56987888
1.0	1.0	0.5	1.27084284	1.27084284
1.0	1.0	0.1	1.43625658	1.43625658

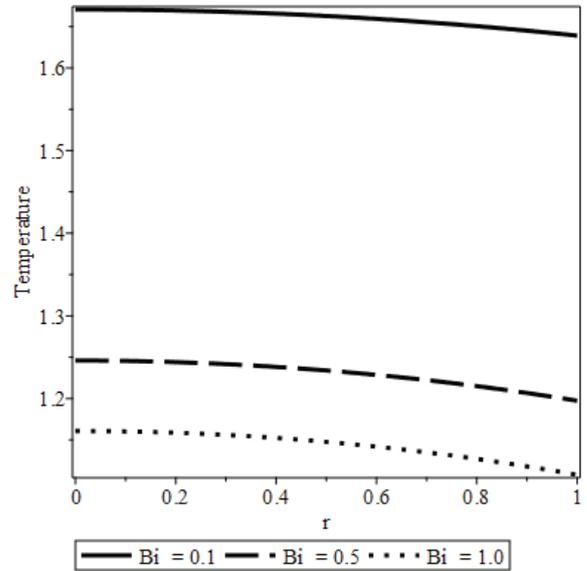


Fig.1. Human head temperature profile for different values of Bi

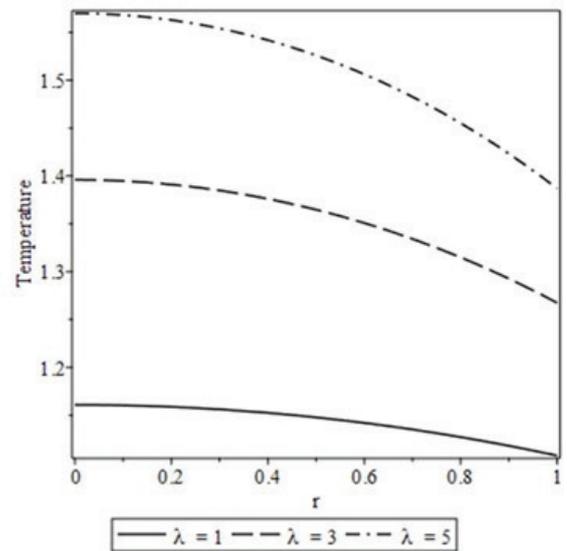


Fig. 2. Human head temperature profile for different values of lambda

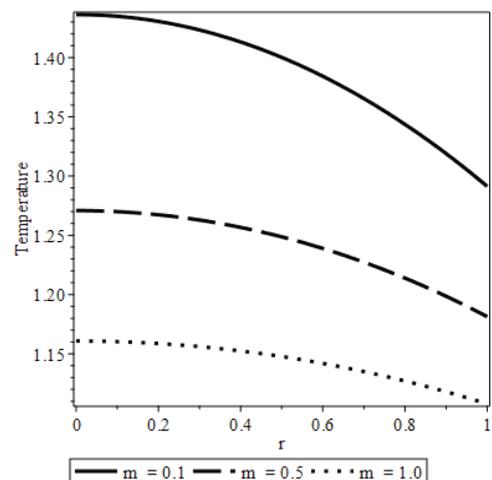


Fig. 3. Human head temperature profile for different values of m

Figure 1 is the Human head temperature profile for different values of biot numbers B_b , the human head temperature is higher at the core and decreases transversely with minimum value at the periphery which agrees with Makinde (2010). An increase in the Biot number due to an increase in the rate of heat-transfer at the interface of human head with the surrounding environment causes a general cooling effect and a reduction in human head temperature. This is in agreement with the earlier results obtained by Çelik and Gokmen(2003) together with Makinde (2010) and Fenuga *et al*(2013).

Figure 2 is the Human head temperature profile for different values of thermogenesis heat production parameter λ . An increase in the rate of thermogenesis heat production parameter λ in a human head resulting from cellular metabolism (as normally experienced during feverish condition) may lead to an elevation in human head temperature. This is also in agreement with the earlier results obtained by Çelik and Gokmen(2003) together with Makinde (2010) and Fenuga *et al* (2013).

Figure 3 illustrates the Human head temperature profile for different values of metabolic thermogenesis slope parameter m . An increase in the

metabolic thermogenesis slope parameter m , may cause a slight decrease in human head temperature which is also in agreement with Makinde (2010).

CONCLUSION

In conclusion, the distribution of the temperature in the human head is very sensitive to environmental temperature changes at the periphery due to convective heat exchange with the ambient temperature. The work shows that diseases like head trauma, stroke, multiple sclerosis, epilepsy, mood disorders and neurodegenerative disorders affect the temperature of the human head. The human head temperature is higher at the core and decreases transversely with the minimum value at the periphery. An increase in Biot number due to an increase in the rate of heat transfer at the interface of the human head with the surrounding environment causes a general cooling effect and a reduction in human head temperature. An increase in the rate of thermogenesis heat parameter resulting from cellular metabolism in the human head may lead to increase in the human head temperature. Finally, an increase in the magnitude of the metabolic thermogenesis slope parameter may cause a slight decrease the temperature of the human head.

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