



LINEAR PROGRAMMING APPROACH TO MODELING FOUNDRY CUPOLA FURNACE CHARGE

*¹Olasunkanmi O A., *¹Hezekiah O. Adeyemi and ²Olaolu Folorunsho.*

*¹Department of Mechanical Engineering, Faculty of Engineering,
Olabisi Onabanjo University, Ago-Iwoye, Nigeria*

*²Department of Computer Engineering, Faculty of Engineering,
Olabisi Onabanjo University, Ago-Iwoye, Nigeria*

**Corresponding Email: oakinyemi@oouagoiwoye.edu.ng*

Manuscript Received: 26/03/2019 Accepted: 11/08/2020 Published: September, 2020

ABSTRACT

In order to control the specifications of the cast iron produced by the cupola furnace, it is necessary to estimate the proportions of the contents of the furnace charge. Traditionally, knowing the percentage of elements present in each constituent of charge and the loss or gain of each element, the final analysis of cast iron can be estimated by trial and error approach - a common approach in most Nigerian foundry workshops. This study presents a programming model technique used to obtain the optimum cupola furnace charge. The objective function was to minimize total cost of raw material and processing costs. The model was developed and implemented using the hypothetical process data of a Foundry workshop. The model was analysed using MATLAB Software Version 7.0. The results obtained using the model is significantly superior to the conventional trial and error approach. It is faster and brings about satisfying a lot of requests at the most minimal expense.

Keywords: *Cupola, Foundry, Furnace charge, Optimization, Linear Programming*

INTRODUCTION

Computer modeling can establish a roadmap for ferrous foundries seeking to achieve process improvement, cost investment funds, and future energy savings. Two consequences of the current economic recession which foundry workshops have begun to take a very close look at are the operating costs and investments in capital equipment improvements. This is key to remain profitable, or at least to minimize losses as much as possible. It is essential to reduce overall costs by becoming more efficient. Fortunately some monetary indicators appear to demonstrate that the economy is beginning to pivot, which could help prepare for expanded spending plans for equipment and other capital enhancements. Numerous cupola shops are swinging to designing investigations as a savvy approach to assess their tasks, and to create vital designs to streamline costs (Abdelrahman and Moore, 2002; David, 2009).

A foundry is a workshop which produces metal casting from either ferrous or non-ferrous alloy. Metals are transformed into parts by liquefying the metal into fluid, pouring the liquid metal in a mould, and after that evacuating the shape materials or casting. The most common metals alloys produced in most Nigerian local foundries are aluminum and cast iron. Be that as it may, other metals, for example, steel, magnesium, copper, tin and zinc can likewise be manufactured. In addition, Furnaces are refractory lined vessels that contain the charge material to be melted. Modern furnaces types include electric-arc furnace, induction furnace, cupola, reverberator and crucible furnaces (Sharma, 2006). Furnace decision is reliant on the alloy system and amounts to be produced. For a cast iron foundry workshop, copula furnace is suitable for the system/workshop.

The issue of deciding the ideal production level at different stages of foundry production is of extraordinary significance. This is so in light of the fact that product blend is essential than product volume and on account of the multifaceted nature of foundry furnace charge, sub-optimal workable solution are generally obtained by experience. Despite the fact that these solutions when actualized accomplish great benefit and better usage however their productivity is not exactly optimal. Linear programming provides a better solution; it has been frequently applied to the problem of finding the best mix of raw materials for a particular selection of products (Salvador and Jose, 2013). Most extensively it is used in business and economic situations (Şenol *et al.*, 2016).

Based on the fact that furnace charge calculation is a raw material mix problem, linear programming has helped in a great deal to make exact and accurate formulation of the problem of content of the charge that is necessary for a particular composition. A lot of studies have been carried out in the area of linear programming applied to foundry operations (Gordana *et al.*, 2008; Abdelrahman

and Moore, 2002; David, 2009; Yang and Park, 2009). These works have extensively discussed the optimization, use of scrap metal in metal production, furnace efficiency and comprehensive energy study and mass balance.

In most cast iron and steel foundry workshop in Nigeria, the copula furnace is one of the furnaces used to melt metal especially cast iron (which contains major chemical elements like C, S, P, Mn and Si). Charging copula furnace for cast iron casting has always been carried out by the method of trial and error. This study adopted a linear programming model to optimize a copula furnace charge in a cast iron foundry workshop situated in Lagos, the Western Nigeria. The objectives are to use the developed model solve an hypothetical process data of a Foundry workshop, compare the result of the model output to the technique of trial and error in terms of speed and cost effectiveness..

MATERIALS AND METHOD

Charge Optimization Methods

The target of charge improvement is to satisfy a lot of requests at the most reduced cost utilizing accessible raw materials. The expenses incorporate the raw material expenses and processing expenses. Charge optimization models typically have limitations for the chemical equalization of the cast iron production, sums and material assets (Xixing *et al.*, 2017; Goutam and Robert, 2001).

These kinds of furnace charge optimization models are available:

1. Single Heat Model: This sort of model considers the subtleties of the procedure. This is on the grounds that the piece stock is generally restricted and limiting raw materials expenses in a single heat can increase the expense of the other heat (Marcus and Lars, 2004).
2. Models, where charge are optimized for several heats. This can consider current raw material stock and anticipate conveyances and offer the accessible assets in optimal way (Kim and Lewis, 1987).
3. Model, where formulas are upgraded for items rather than heats (Sandberg, 2005). This methodology makes it less demanding to define significant lot models. It forms the basis of the model used in the study.

The ideal recipe for copula charge would yield accurate element concentration or compositions. For some elements this implies the concentrations fulfill the lower limits of the product's standard or are close to them. A disappointment happens when the components substance of cast iron surpasses the upper limits of the product standard since it is normally hard to expel any parts from the molten cast iron (Sudhakar, 2012). It is in this manner imperative that components in the molten cast iron don't surpass

the standard upper limits. The lower limits are not basic on the grounds that the component substance can be expanded however all things considered the processing time is expanded. Additionally raw materials cost will be higher in light of the fact that alloys are more costly than scrap (Preston *et al.*, 2005).

$$mt_{i(min)} \leq \sum_{j=1}^n \sum_{i=1}^b mp_{ij}x_j \leq mt_{i(max)}$$

$$\sum_{j=1}^n x_j \geq 0$$

The Model Development

The model minimizes raw material costs and processing costs. The raw material costs include cost of purchasing raw materials (which include pig iron, scrap, returns, coke and fluxes). The processing costs are entirely the cost of melting down the cupola charge. The model considers one request at once and the goal is to make amount (Kg) of desired products. The objective function is the criterion the decision-maker (melter) use to assess alternative solutions to the problem. The objective function is to minimize total cost given by:

$$Min \sum_{j=1}^n C_j x_j \dots \dots \dots 1$$

The decision variables are components under the control of the decision – maker and their qualities decide the arrangement of the model. The requirements of the problem are the restriction imposed upon the values of the decision variables by the characteristics of the problem. The model assumes that the workshop does not want to keep any casting on hand; hence the exact amount ordered is produced:

$$\sum_{j=1}^n m_j = m \dots \dots \dots 2$$

The restrictions regarding percentage compositions of elements in raw material are represented by inequalities. The compositions of elements in the product are limited between the lower and upper bounds given by:

$$mt_{i(min)} \leq \sum_{j=1}^n \sum_{i=1}^b mp_{ij}x_j \leq mt_{i(max)} \dots \dots \dots 3$$

It should be noted that constituent elements presents in a cast iron foundry workshop are C, Si, Mn, P, S and the base element Fe.

Finally, there is the obvious non-negative constraint:

$$\sum_{j=1}^n x_j \geq 0 \dots \dots \dots 4$$

The linear programming model now expressed as:

$$Min \sum_{j=1}^n C_j x_j$$

Subject to:-

$$\sum_{j=1}^n mx_j = m$$

i=1,2,3,...,b
j=1,2,3,...,n

Nomenclatures and Symbols

Below are the symbols used in this study:

- c_j –Cost (Processing cost and Raw material cost of raw material j) in N.
- m – Amount/mass of product (casting) in kg.
- P_{ij} – % composition of element i in raw material j.
- x_j – Decision variable, charge of raw material j in kg.
- t_i (min) – Lower bound of element i composition in product.
- t_i (max) – Upper bound of element i composition in product.
- n – Raw material counters.
- b – Element counters.

Model Application

A hypothetical order of a firm was considered using the Linear Programming model method developed. A cast Iron foundry shop with has a company’s order to manufacture 200’000 kg of castings containing at least 0.45% Manganese and between 3.25% and 5.50% Silicon. The castings sold for N4500 per kg. The foundry shop had three kinds of pig iron accessible in basically boundless sums, with the accompanying properties:

Table 1: Composition of available pig iron

Types of Pig Iron	A	B	C
Silicon	4%	1%	0.6%
Manganese	0.45%	0.5%	0.4%

The manufacture processes was to such an extent that unadulterated manganese can likewise be added to the melt. The expenses of different conceivable inputs were:

- Pig Iron A= N 21’000 / thousand kg
- Pig Iron B= N 25’000 / thousand kg
- Pig Iron C= N 15’000 / thousand kg
- Manganese= N 8000/kg

It costed N5 to melt down a kg of pig iron. The linear programming model for this problem is given below:-

$$Total\ Income = 2 \times 10^5 \times 4500 = N9 \times 10^8$$

- x₁ = thousand of kg of Pig Iron A
- x₂ = thousand of kg of Pig Iron B
- x₃ = thousand of kg of Pig Iron C
- x₄ = kg of unadulterated manganese

To determine the all-out expenses brought about underway of production of the alloy, the melting cost

in addition to cost of each kind of pig iron utilized is
 Pig Iron A = 21,000 + 5000 = 26,000
 Pig Iron B = 25,000 + 5000 = 30,000
 Pig Iron C = 15,000 + 5000 = 20,000
 Total cost = 26,000x₁ + 30,000x₂ + 20,000x₃ + 8,000x₄
 Total Profit = 9×10⁸-26,000x₁ -30,000x₂ - 20,000x₃ - 8,000x₄

Constraints: Restriction imposed upon the estimates of the decision variables by qualities of foundry charge problem at hand:

Exact ordered amount:

$$\sum_{j=1}^{n_4} m_j x_j = m$$

$$\sum_{j=1}^{n_4} 200000 x_j = 200000$$

$$\sum_{j=1}^{n_4} 200000 x_1 + 200000 x_2 + 200000 x_3 + 200000 x_4 = 200000$$

Manganese Constraint:

$$(200000 \times 0.0045 x_1) + (200000 \times 0.005 x_2) + (200000 \times 0.004 x_3) + x_4 \geq 200000 \times 0.0045$$

$$900x_1 + 1000x_2 + 800x_3 + x_4 \geq 900$$

3. Silicon Constraint:

$$(200000 \times 0.04 x_1) + (200000 \times 0.01 x_2) + (200000 \times 0.006 x_3) \geq 6500$$

$$8000x_1 + 2000x_2 + 1200x_3 \geq 6500$$

and

$$(200000 \times 0.04 x_1) + (200000 \times 0.01 x_2) + (200000 \times 0.006 x_3) \leq 11000$$

$$8000x_1 + 2000x_2 + 1200x_3 \leq 11000$$

4. Non-Negative Constraint:

$$\sum_{j=1}^4 x_j \geq 0$$

The objective function can either be maximized or minimized. So, in the event that we minimize the total cost, the subsequent linear programming (LP) problem is presented below:

Minimize: Z = 26000x₁ + 30000x₂ + 20000x₃ + 80000x₄

Subject to:

$$200,000x_1 + 200,000x_2 + 200,000x_3 + x_4 = 200,000$$

$$900x_1 + 1000x_2 + 800x_3 + x_4 \geq 900$$

$$8000x_1 + 2000x_2 + 1200x_3 \geq 6,500$$

$$8000x_1 + 2000x_2 + 1200x_3 \leq 11,000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The above linear programming model was solved using MATLAB Software Version 7.0 (2007) and the solution presented below:

```
>> clear f A b
```

```
f = [26000 30000 20000 8000];
```

```
A = [200000 200000 200000 1; -200000 -200000 -200000 -1; -900 -1000 -800 1; -8000 -2000 -1200 0; 8000 2000 1200 0; 1 0 0 0; 0 -1 0 0; 0 0 -1 0; 0 0 0 -1; -1 0 0 0];
```

```
b = [200000; -200000; -900; -6500];
```

```
11000; 0; 0; 0; 0; 0];
```

```
simpl (f,A,b)
```

```
answer =
```

```
0.7656
```

```
0.1172
```

```
0.1172
```

```
0
```

Therefore, for a 1000 kg product mix, the amount of each raw material is given below:

Pig Iron A = 765.6 kg

Pig Iron B = 117.2 kg

Pig Iron C = 117.2 kg

Manganese = 0 kg

RESULTS AND DISCUSSION

The model was tested using MATLAB software Version 7.0 with hypothetical process data of a Foundry workshop. The data used as stated in the methodology included only raw material composition and cost as the real processing costs were not available. The methodology described the linear programming (LP) model developed for a typical furnace charge. Results showed that there was no waste of time during the use of the LP model. The results obtained are in qualitative and quantitative agreement with previous work done (Abdelrahman and Moore, 2002; David, 2009).

By using linear programming model to solve charge calculation, effort have been made to reduce time spent and cost of production in the foundry workshop; effort have also been made to eliminate the process of trial and error, a process that is still used in different foundries in the country. With the potential cost benefits in energy and operations as a result implementing the developed LP model, it ought to be moderately simple for most foundries to justify the relative minimal cost of this model, and to venture cost benefits and improved effectiveness. The aftereffects of this research consider unequivocal, long term planning by administrators, melters and engineers alike; to guarantee future optimization and cost benefits that will improve the focused position of the foundries that venture into implementing the developed LP model.

CONCLUSION

This study developed a linear programming model to obtain the optimum cupola furnace charge. The model simultaneously minimizes raw material, cost and save a lot of time which can be channeled to other production activities. The model has been tested for optimal pig iron grade allocation. Numerical results showed that pig iron can be utilized more effectively, essential raw materials can be saved and expenses can be brought down by utilizing right pig iron inputs in right raw material mix. The model will help foundry workshop decision makers to obtain the best raw materials mix thereby eliminating wastage and the adjustments in the conventional trial and error approach.

REFERENCES

- Abdelrahman, M. A. and Moore, K. L. (2002). Optimal choice of cupola furnace nominal operating point. Proceedings of the Thirtieth Southeastern Symposium, Morgantown, WV, USA. Pp. 496 – 499.
- David J. K. (2009). Engineering studies for cupola melt shops. Foundry Management and Technology, Penton Media Inc., USA.
- Gordana M., Niko M. and Tadija L. (2008) Production Scheduling Model in Aluminium Foundry *Journal of Mechanical Engineering*. 54(1):37-48
- Goutam D. and Robert F. (2001) A Survey of Mathematical Programming Applications in Integrated Steel Plants. 2001 Goutam Dutta and Robert Fourer. <http://users.iems.northwestern.edu/~4er/STEEL/survey.html>
- Kim, J. and Lewis, R.L. (1987). A large scale linear programming application to least cost charging for foundry melting operations. AFS Transactions.
- Marcus K. and Lars E. (2004) A Specific Heat Ratio Model for Single-Zone Heat Release Models. Society of Automotive Engineers, Inc. http://www.vehicular.isy.liu.se/Publications/Articles/SAE_04_MKLE.pdf
- Preston L., Sigrid G., Hans O.R. and Randolph K. (2005). Identifying Economic and Scrap reuse benefits Of Light Metals Sorting Technologies. EPD Congress of The Minerals, Metals & Materials Society, 2005 http://msl.mit.edu/publications/Sorting_EPD_Final_Li.pdf
- Salvador G. M. and Jose M. (2013). Optimal Selection of Raw Materials for Pharmaceutical Drug Product Design and Manufacture using Mixed Integer Nonlinear Programming and Multivariate Latent Variable Regression Models. *Ind. Eng. Chem. Res.*, 2013, 52 (17):5934–5942
- Sandberg E. (2005). Energy and Scrap Optimisation of Electric Arc Furnaces by Statistical Analysis of Process Data. Available from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.399.4857&rep=rep1&type=pdf>
- Şenol A., Seyyide, D. and Hediye İ. (2016). Maximization of National Income with Linear Programming and Input - Output Analysis: *An Application for Turkey Journal of Business & Economic Policy*, 3(2): 29.
- Sharma, P.C. (2006). A Textbook of Production Technology (Manufacturing Processes). Schand and Co. Ltd., New Delhi Revised Edition.
- Sudhakar K.V. (2012) Cast iron component failure : A metallurgical investigation. *Archives of foundry engineering*. 12 (2):67–70.
- Xixing L., Shunsheng G., Yi L., Baigang D. and Lei W. (2017). A Production Planning Model for Make-to-Order Foundry Flow Shop with Capacity Constraint. *Mathematical Problems in Engineering Vol. 2017, ID 6315613, 15 pages.* <https://doi.org/10.1155/2017/6315613>
- Yang, J-M Park Y K (2009) Scheduling of casting in real foundries using linear programming (2009). Proceedings of the Institution of Mechanical Engineers, Part B: *Journal of Engineering Manufacture*. 223(10):1351 – 1360.